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Zeeman Effect Lab

Purpose & Introduction:

In this lab, we will determine the Bohr Magneton through the exploration of the Zeeman Effect. The Zeeman effect is the splitting of emission lines when the emitting source is placed in a magnetic field. In our case, we will place Mercury in a magnetic field and look at the emission line splitting of the 546.07 nm line.

The Zeeman effect is caused by the splitting of energy levels with different angular momenta in the atomic structure. The electrons with non-zero angular momenta will have a non-zero magnetic dipole moment that will interact with the external magnetic field. This interaction will change the energy of the electron.

In the case of Mercury, two electrons fill the 6s shell. When Mercury is excited to the point of discharge, these electrons will undergo several different possible transitions as illustrated in figure 1.

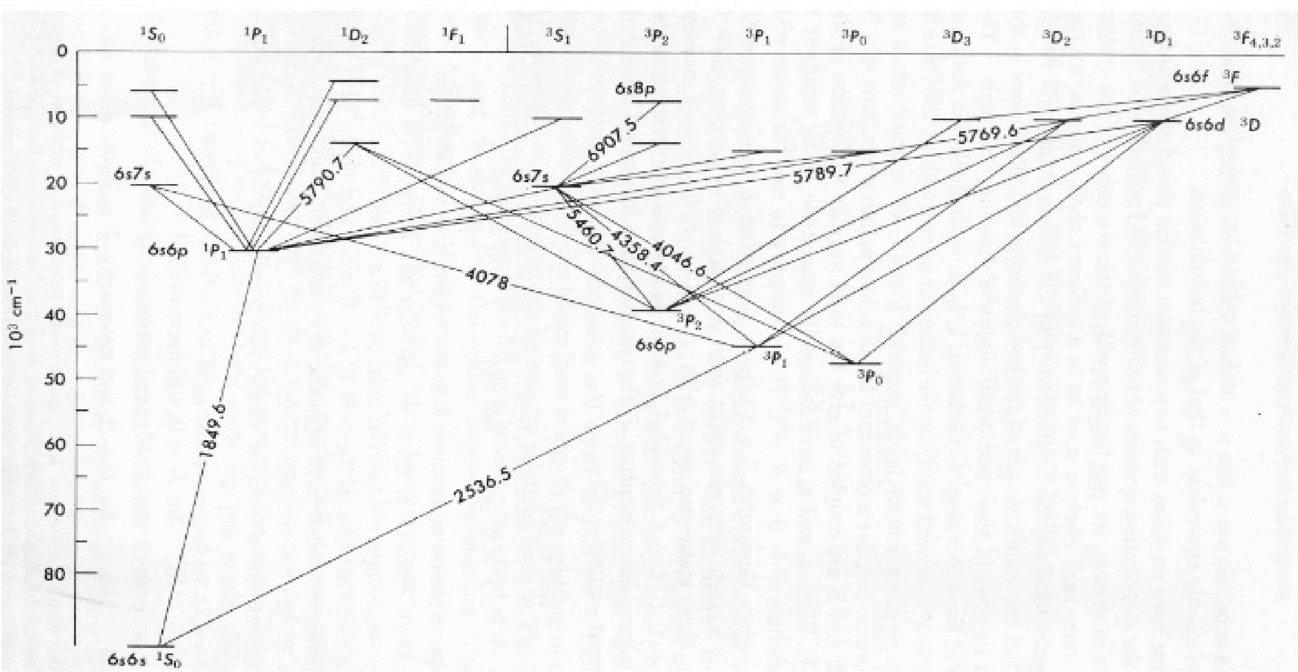

Figure 1: This picture illustrates the possible transition of the electrons originally found in the 6s shell. One of these electrons can transition to the 7s shell and from there down to the 6p shell. AS illustrated there are many possible transitions. However, we will be filtering out the 5460.7 angstrom line, the green line, using a narrow pass-band filter called an interference filter. (All transitions are measured in Angstroms.)

In our experiment we will filter our the 546.07 nm line. Originally the Mercury goes through a transition from the 6s to the 7s energy level. The electron pair sharing the 6s energy level are spin up and spin down with orbital angular momentum (denoted by the quantum number *l*) of zero since they are in the s energy level. However, when one of these electrons transitions to the 7s state its spin changes to be the same as the spin of the electron that remains in the 6s energy level. This yields a combined spin state of 1 since each electron has a spin of ½ (spin quantum number denoted with *s*). The total angular momentum is denoted by the quantum number *j* which is a combination of *s* and *l*. Therefore, the 6s to 7s combined state has j=1, s=1, and l=0. The z-component of the angular momentum is taken to be along the axis of the magnet, thus aligned with the magnetic field lines. This component is represented by m_j and has possible values of $-j,-j+1,...,j-1,j$. The energy contained in these possible states is equal unless there is an external magnetic field present. In this case, the states are no longer degenerate and now each have different energies. In the 6s,7s state, there are three possible values of m_i:$-1,0,1$. In the presence of a magnetic field these states differ in energy; otherwise, they are equal in energy. In the 6s,6p transition state j=2, s=1, and l=1. This yields 5 m_j states (-2,-1,0,1,2). Now, in our lab, we will observe the transition of the electron in the 7s state to the 6p state to create a new combined system. When the electrons transit between these sates in the presence of a magnetic field we will observe a splitting of the 546.07 nm line because the m_j states now have different energies. From the selection rule no transitions of $|\Delta m_j|>1$ are present. Therefore, 9 lines will be present in the presence of a magnetic field. The transitions are illustrated in figure 3. Also present in figure 2 are the Lande g-factors for the states of the combined 6s, 7s and 7s, 6p. This factor governs the spacing between the m_j levels. It is given by the following equation:

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}.$$

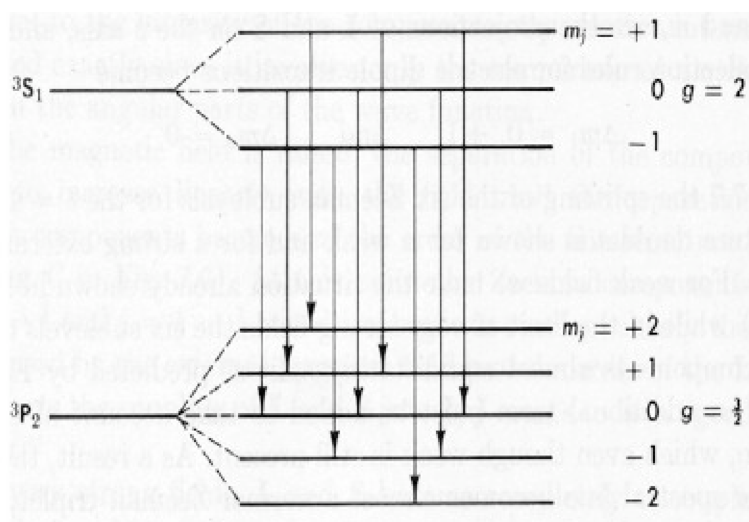

Figure 2: In this diagram we can see the possible transitions for the splitting of the 546.07 nm line. The first three from the left and the last three from the right are the σ-lines which are circularly polarized (but appear linearly polarized from the direction the camera observes in) and the middle three lines are the π-lines and are planar polarized along the direction of the magnetic field lines.

In order to determine the Bohr magneton we will measure the differences in energy between the transitions represented by the 9 split lines in the spectrum we observe. The differences in energy will allow us to find the Bohr magneton as shown in equation (1) (where B is the magnetic field, μ_0 is the Bohr magneton, *g* is the Lande g-factor of the 6s,6p combined state, and m_j is the z-component of angular momentum in the state to which the electron transitioned.

$$\Delta E = g\mu_0 B m_j \tag{1}$$

The Bohr magneton is also equivalent to equation (2) (where m_e is the mass of the electron and e is the elementary charge)

$$\mu_0 = \frac{e\hbar}{2m_e} \tag{2}$$

In order to find the differences in energy between the states being transitioned to, we must derive an equation for the change in frequency between these lines. This will require an explanation of the Fabri-Perot Cavity being used in our experiment. The Fabri-Perot cavity consists of 2 semi-transmissive mirrors a fixed distance apart (in our case 2 mm). The cavity is non-transimissive to light light incident on it unless the distance between the mirrors is an integer number of wavelengths of the incident light. In the case that it is, a standing wave will form inside the cavity. But, as the cavity mirrors are semi-transmissive, the light will be emitted from the cavity. In fact, it will be approximately the same intensity as the incident light to the cavity because the standing wave formed inside the cavity is built from constructively interfering light, thus intensifying the incident light. For small angles of incidence, we find that the light will inter the cavity, form a standing wave, and exit if the above criterion is satisfied. The following picture illustrates light entering a Fabri-Perot cavity.

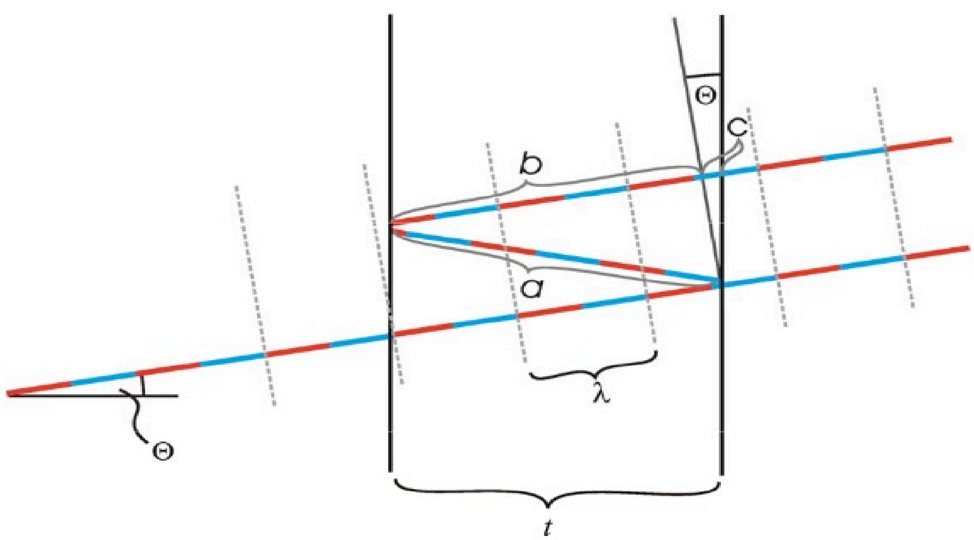

Figure 3: In this picture, incident light of angle θ is enters a cavity of width t and forms a mostly overlapping standing wave.

In this case if the sum of *a* and *b* are equivalent to an integer number of wavelengths, then such incident light will be allowed to pass through the cavity.

We find that the following equation from figure 3.

$$2t\cos(\theta_n) = n\lambda \tag{3}$$

From equation (3) it is apparent that several possible angles exist for light that passes through the cavity. In fact, there will be several orders of rings. The rings, measured from the center starting with 0, will be denoted by *p* and the order of the ring shall be denoted by n_p. n_0 will denote the number of whole wavelengths in a round trip through the cavity at normal incidence.

$$n_p = n_0 - p \tag{4}$$

However, it is unlikely that normally incident light will have an integer number of wavelengths fill the round trip distance in the cavity. Therefore, we denote ε as the fractional wavelength left over during a round trip at normal incidence.

$$2t = (n_o + \varepsilon)\lambda \tag{5}$$

We can use the small angle approximation to rewrite the transmission criteria

$$\begin{gathered} 2t\cos(\theta_n) = n\lambda \\ \Downarrow \\ 2t\cos(\theta_p) = (n_0 - p)\lambda \\ \Downarrow \\ 1 - \frac{\theta_p^{\,2}}{2} = \frac{(n_0 - p)\lambda}{2t} \end{gathered} \tag{6}$$

We can then rewrite $n_0 - p$.

$$n_o - p = (n_0 + \varepsilon) - p - \varepsilon = \frac{2t}{\lambda} - p - \varepsilon \tag{7}$$

We can then substitute (7) into (6) and solve for θ_p .

$$\theta_p = \frac{\lambda}{2t}\sqrt{p + \varepsilon} \tag{8}$$

Given the radius of curvature of the incident beam of light being $\mathcal{R}$, we can define the radii of the concentric circles we will observe in the emission spectra.

$$r_p = \mathcal{R}\tan(\theta_p) \approx \theta_p = \frac{\mathcal{R}\lambda}{2t}\sqrt{p + \varepsilon} \tag{9}$$

In the above equation $\frac{\mathcal{R}\lambda}{2t}$ is a constant; therefore, we can rewrite equation (9).

$$r_p = C\sqrt{p+\varepsilon} \tag{10}$$

We can fit the our data as r_p *vs.* p using ε and C as fit parameters.

$$\varepsilon = \frac{2t}{\lambda} - n_0 \tag{11}$$

If two of the wavelengths are (the split emission lines wavelengths) are extremely close, then we can assume that they have the same n_0 and thus we can calculate there differences as follows.

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{2t}(\varepsilon_1 - \varepsilon_2) \tag{12}$$

The difference in energy between these wavelengths is given by:

$$\Delta E = \frac{ch}{2t}(\varepsilon_1 - \varepsilon_2) \tag{13}$$

<u>The Set-up</u>

Our set-up is depicted in the following picture:

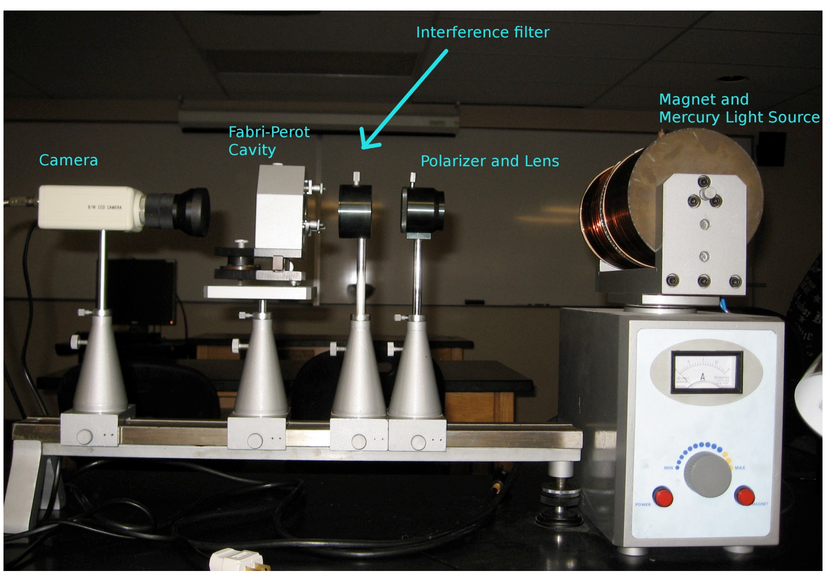

Figure 4: This is the set-up of the experiment. Each of the parts has been explained except for the polarizer. This allows us to select the σ-lines and π-lines separately.

First we aligned the above parts along their track so that we obtained a clear image from the camera. The image we obtained is the following without any external magnetic field applied.

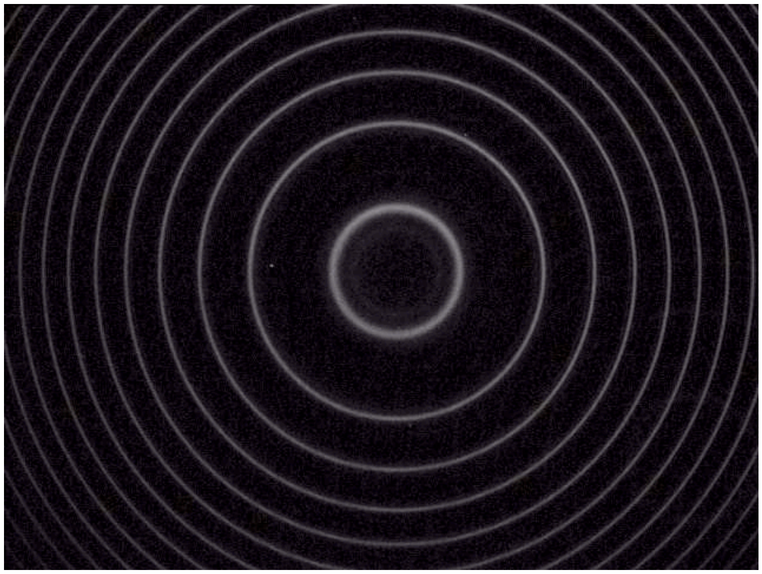

Figure 5: This is a picture of the 546.07 nm emission line of mercury without a magnetic field applied.

Next we turned on the magnetic field and separately displayed the σ -lines and π -lines.

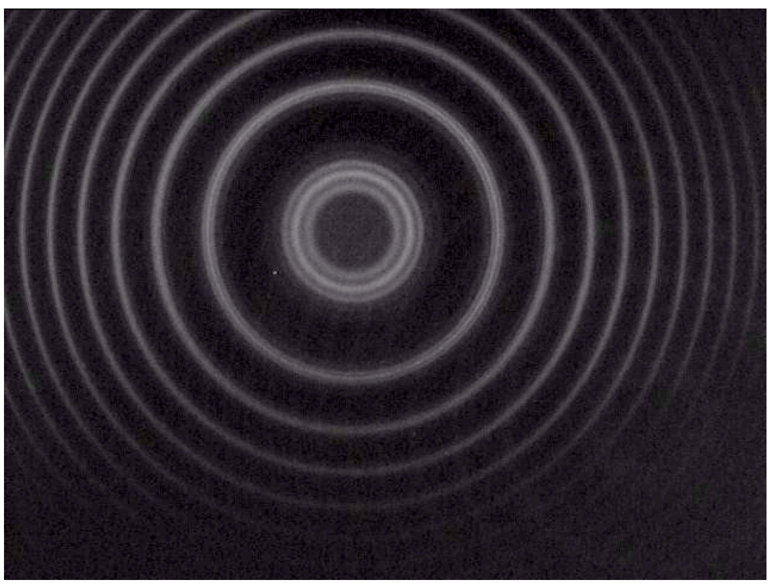

Figure 6: This picture illustrates the π-lines.

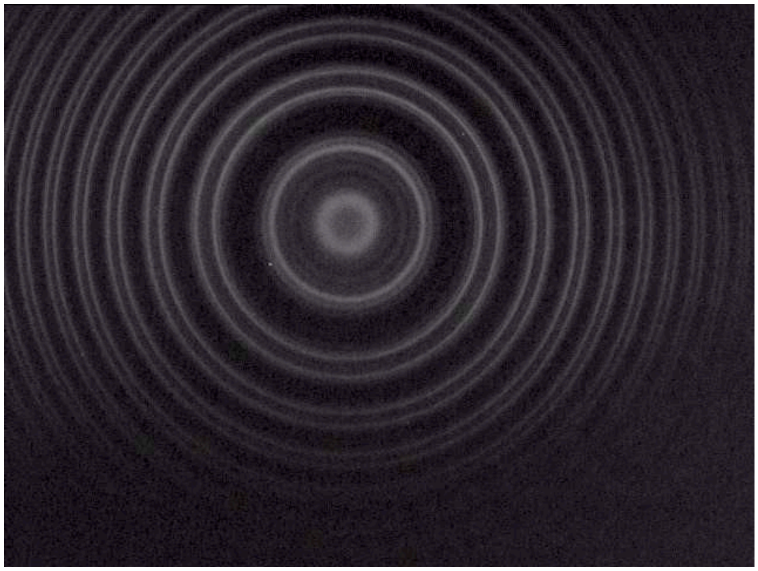

Figure 7: This picture illustrates the σ-lines.

After obtaining the above photographs, we began systematically taking photographs of the split lines. However, it must be noted that we were only able to take measurements of the brightest sigma and pi lines. Furthermore, we were only able to take measurements of the 0^{th}, 1^{st}, and 2^{nd} order ring groups. Each of these groupings contained 3 lines, 2 sigma lines and 1 pi line in between the sigma lines. These lines corresponded to the middle transitions in the three groups of lines in figure 2. We took measurement for varying currents, which changes the magnetic field. We took measurements at .49 amps and then .6 amps and every tenth of an amp after that up to 1.3 amps. We were careful not to remain above .7 amps for more than 10-15 seconds to prevent damage to the magnet.

We measured the three split lines using the program that runs the Zeeman effect camera. It allows us to save pictures from the camera as JPG files. Then, we can can manipulate the pictures to varying degrees of gray scale in order to better see the lines. After the lines are in focus, we draw circles corresponding to the concentric circles we see in the photographs in order to measure the radius of each of the circles in the first 3 inner groupings of lines. Finally we fit our data and analyzed it to find the Bohr Magneton.

The following are examples of the photographs we measured the ring from at varying currents.

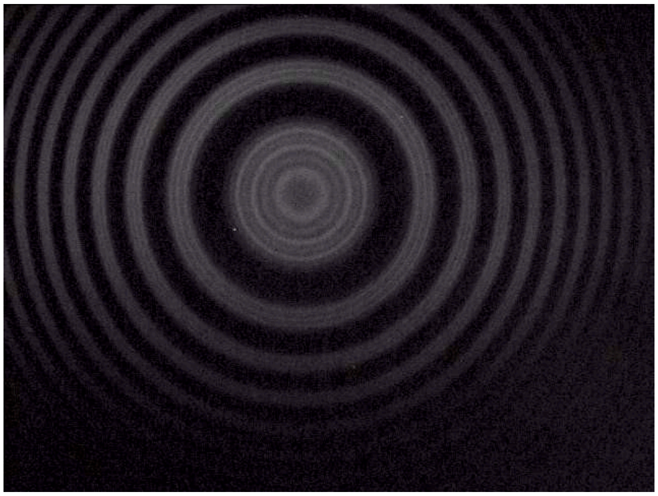
Figure 8: Rings with applied current of .49 Amps

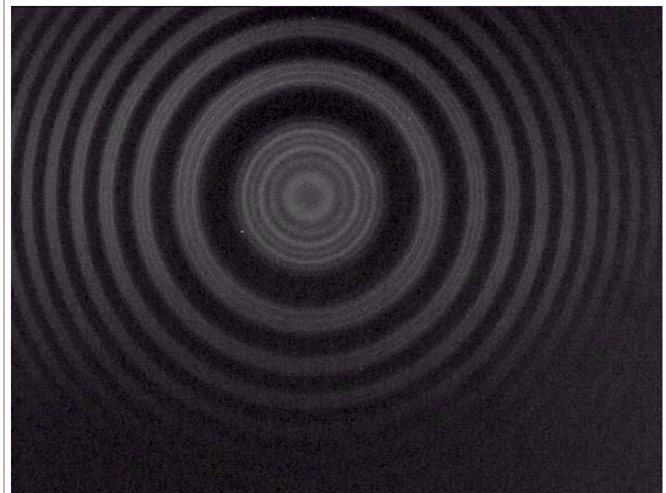
Figure 9: Rings with applied current of .6 Amps

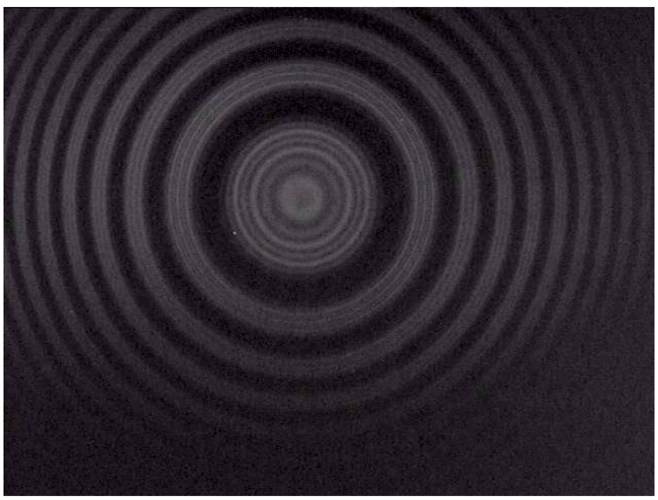
Figure 10: Rings with applied current of .7 Amps

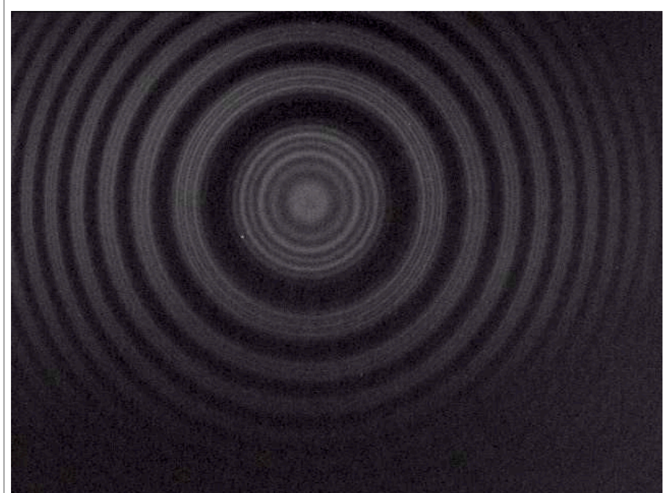
Figure 11: Rings with applied current of .8 Amps

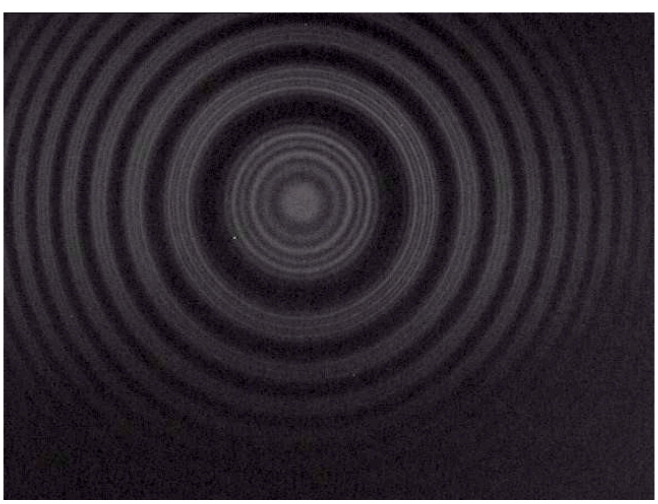
Figure 12: Rings with applied current of .9 Amps

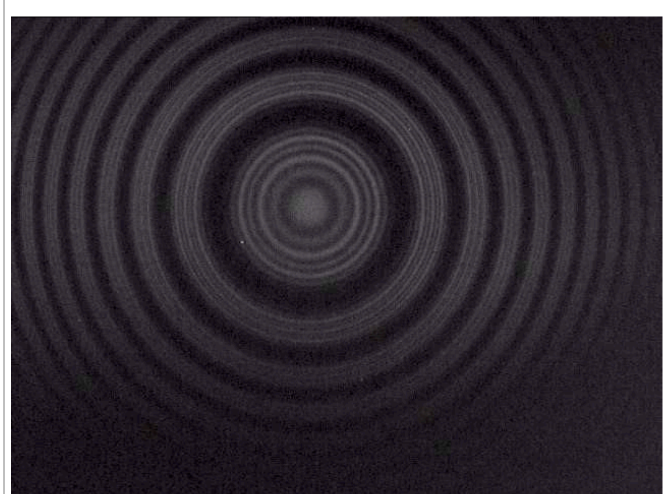
Figure 13: Rings with applied current of 1.0 Amps